2. Measure the angle of tilt relative to the horizontal and find μ_k .

GRASP CHECK

True or False—If only the angles of two vectors are known, we can find the angle of their resultant addition vector.

a. True
b. False

Check Your Understanding

17. What is friction?

a. Friction is an internal force that opposes the relative motion of an object.
b. Friction is an internal force that accelerates an object's relative motion.
c. Friction is an external force that opposes the relative motion of an object.
d. Friction is an external force that increases the velocity of the relative motion of an object.

18. What are the two varieties of friction? What does each one act upon?

a. Kinetic and static friction both act on an object in motion.
b. Kinetic friction acts on an object in motion, while static friction acts on an object at rest.
c. Kinetic friction acts on an object at rest, while static friction acts on an object in motion.
d. Kinetic and static friction both act on an object at rest.

19. Between static and kinetic friction between two surfaces, which has a greater value? Why?

a. The kinetic friction has a greater value because the friction between the two surfaces is more when the two surfaces are in relative motion.
b. The static friction has a greater value because the friction between the two surfaces is more when the two surfaces are in relative motion.
c. The kinetic friction has a greater value because the friction between the two surfaces is less when the two surfaces are in relative motion.
d. The static friction has a greater value because the friction between the two surfaces is less when the two surfaces are in relative motion.

5.5 Simple Harmonic Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Hooke's law and Simple Harmonic Motion
- Describe periodic motion, oscillations, amplitude, frequency, and period
- Solve problems in simple harmonic motion involving springs and pendulums

Section Key Terms

amplitude	deformation	equilibrium position	frequency
Hooke's law	oscillate	period	periodic motion
restoring force	simple harmonic motion	simple pendulum	

Hooke's Law and Simple Harmonic Motion

Imagine a car parked against a wall. If a bulldozer pushes the car into the wall, the car will not move but it will noticeably change shape. A change in shape due to the application of a force is a **deformation**. Even very small forces are known to cause some deformation. For small deformations, two important things can happen. First, unlike the car and bulldozer example, the object returns to its original shape when the force is removed. Second, the size of the deformation is proportional to the force. This second property is known as **Hooke's law**. In equation form, Hooke's law is

$$\mathbf{F} = -\mathbf{kx},$$

where $\mathbf{x}$ is the amount of deformation (the change in length, for example) produced by the restoring force $\mathbf{F}$, and $\mathbf{k}$ is a constant that depends on the shape and composition of the object. The restoring force is the force that brings the object back to its equilibrium position; the minus sign is there because the restoring force acts in the direction opposite to the displacement. Note that the restoring force is proportional to the deformation $\mathbf{x}$. The deformation can also be thought of as a displacement from equilibrium. It is a change in position due to a force. In the absence of force, the object would rest at its equilibrium position. The force constant $\mathbf{k}$ is related to the stiffness of a system. The larger the force constant, the stiffer the system. A stiffer system is more difficult to deform and requires a greater restoring force. The units of $\mathbf{k}$ are newtons per meter (N/m). One of the most common uses of Hooke's law is solving problems involving springs and pendulums, which we will cover at the end of this section.

Oscillations and Periodic Motion

What do an ocean buoy, a child in a swing, a guitar, and the beating of hearts all have in common? They all **oscillate**. That is, they move back and forth between two points, like the ruler illustrated in Figure 5.37. All oscillations involve force. For example, you push a child in a swing to get the motion started.

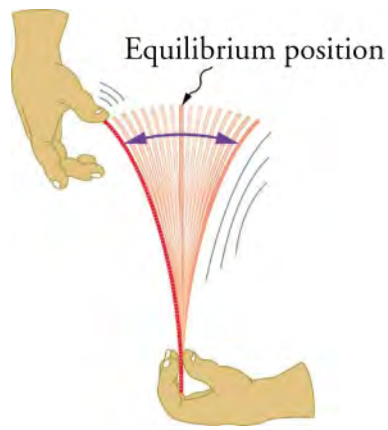

Figure 5.37 A ruler is displaced from its equilibrium position.

Newton's first law implies that an object oscillating back and forth is experiencing forces. Without force, the object would move in a straight line at a constant speed rather than oscillate. Consider, for example, plucking a plastic ruler to the left as shown in Figure 5.38. The deformation of the ruler creates a force in the opposite direction, known as a **restoring force**. Once released, the restoring force causes the ruler to move back toward its stable equilibrium position, where the net force on it is zero. However, by the time the ruler gets there, it gains momentum and continues to move to the right, producing the opposite deformation. It is then forced to the left, back through equilibrium, and the process is repeated until it gradually loses all of its energy. The simplest oscillations occur when the restoring force is directly proportional to displacement. Recall that Hooke's law describes this situation with the equation $\mathbf{F} = -\mathbf{kx}$. Therefore, Hooke's law describes and applies to the simplest case of oscillation, known as **simple harmonic motion**.

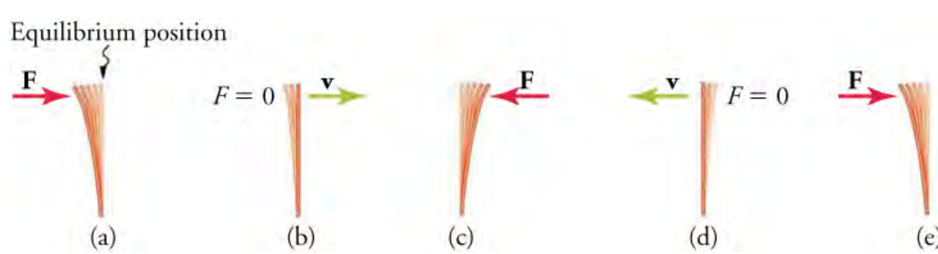

Figure 5.38 (a) The plastic ruler has been released, and the restoring force is returning the ruler to its equilibrium position. (b) The net force is zero at the equilibrium position, but the ruler has momentum and continues to move to the right. (c) The restoring force is in the opposite direction. It stops the ruler and moves it back toward equilibrium again. (d) Now the ruler has momentum to the left. (e) In the absence of damping (caused by frictional forces), the ruler reaches its original position. From there, the motion will repeat itself.

When you pluck a guitar string, the resulting sound has a steady tone and lasts a long time. Each vibration of the string takes the same time as the previous one. **Periodic motion** is a motion that repeats itself at regular time intervals, such as with an object bobbing up and down on a spring or a pendulum swinging back and forth. The time to complete one oscillation (a complete cycle of motion) remains constant and is called the **period** T. Its units are usually seconds.

Frequency f is the number of oscillations per unit time. The SI unit for frequency is the hertz (Hz), defined as the number of oscillations per second. The relationship between frequency and period is

$$f = 1/T.$$

As you can see from the equation, frequency and period are different ways of expressing the same concept. For example, if you get a paycheck twice a month, you could say that the frequency of payment is two per month, or that the period between checks is half a month.

If there is no friction to slow it down, then an object in simple motion will oscillate forever with equal displacement on either side of the equilibrium position. The **equilibrium position** is where the object would naturally rest in the absence of force. The maximum displacement from equilibrium is called the **amplitude X**. The units for amplitude and displacement are the same, but depend on the type of oscillation. For the object on the spring, shown in Figure 5.39, the units of amplitude and displacement are meters.

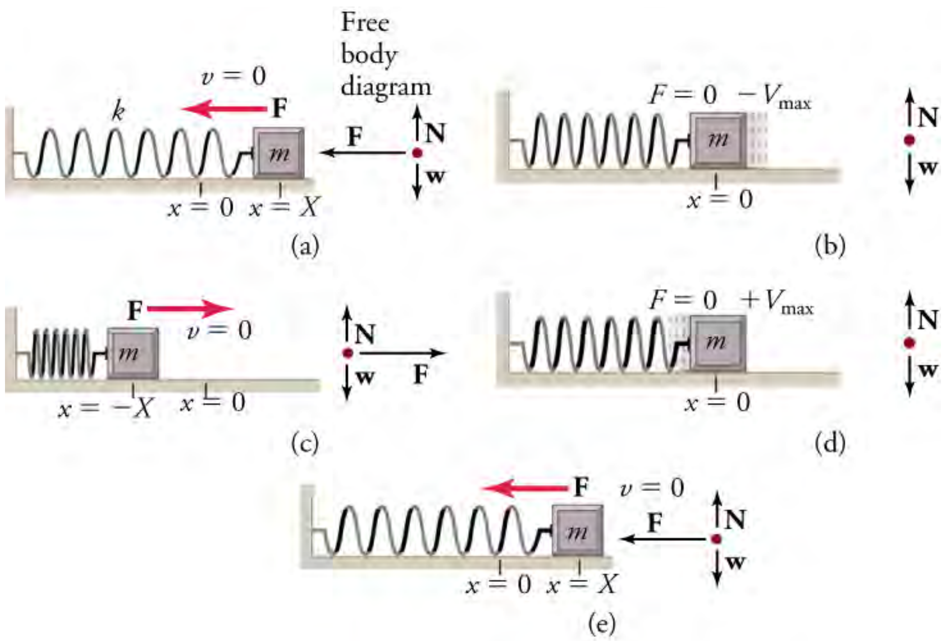

Figure 5.39 An object attached to a spring sliding on a frictionless surface is a simple harmonic oscillator. When displaced from equilibrium, the object performs simple harmonic motion that has an amplitude **X** and a period T. The object's maximum speed occurs as it passes through equilibrium. The stiffer the spring is, the smaller the period T. The greater the mass of the object is, the greater the period T.

The mass m and the force constant **k** are the *only* factors that affect the period and frequency of simple harmonic motion. The period of a simple harmonic oscillator is given by

$$T = 2\pi\sqrt{\frac{m}{\mathbf{k}}}$$

and, because $f = 1/T$, the frequency of a simple harmonic oscillator is

$$f = \frac{1}{2\pi}\sqrt{\frac{\mathbf{k}}{m}}.$$

WATCH PHYSICS

Introduction to Harmonic Motion

This video shows how to graph the displacement of a spring in the x-direction over time, based on the period. Watch the first 10 minutes of the video (you can stop when the narrator begins to cover calculus).

Click to view content (https://www.khanacademy.org/embed_video?v=Nk2q-_jkJVs)

GRASP CHECK

If the amplitude of the displacement of a spring were larger, how would this affect the graph of displacement over time? What would happen to the graph if the period was longer?

a. Larger amplitude would result in taller peaks and troughs and a longer period would result in greater separation in time between peaks.
b. Larger amplitude would result in smaller peaks and troughs and a longer period would result in greater distance between peaks.
c. Larger amplitude would result in taller peaks and troughs and a longer period would result in shorter distance between peaks.
d. Larger amplitude would result in smaller peaks and troughs and a longer period would result in shorter distance between peaks.

Solving Spring and Pendulum Problems with Simple Harmonic Motion

Before solving problems with springs and pendulums, it is important to first get an understanding of how a pendulum works. Figure 5.40 provides a useful illustration of a simple pendulum.

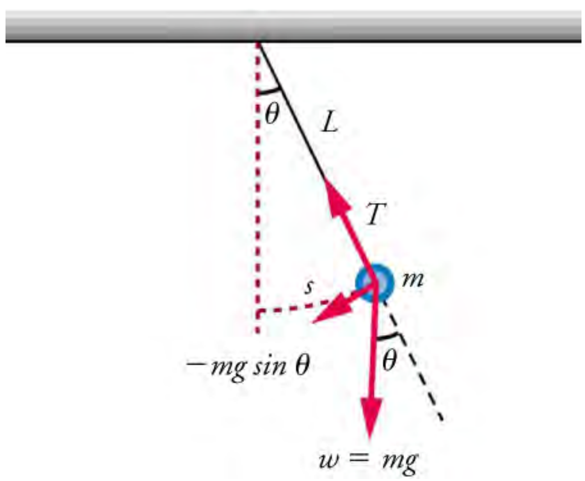

Figure 5.40 A simple pendulum has a small-diameter bob and a string that has a very small mass but is strong enough not to stretch. The linear displacement from equilibrium is s, the length of the arc. Also shown are the forces on the bob, which result in a net force of $-mg \sin\theta$ toward the equilibrium position—that is, a restoring force.

Everyday examples of pendulums include old-fashioned clocks, a child's swing, or the sinker on a fishing line. For small displacements of less than 15 degrees, a pendulum experiences simple harmonic oscillation, meaning that its restoring force is directly proportional to its displacement. A pendulum in simple harmonic motion is called a **simple pendulum**. A pendulum has an object with a small mass, also known as the pendulum bob, which hangs from a light wire or string. The equilibrium position for a pendulum is where the angle θ is zero (that is, when the pendulum is hanging straight down). It makes sense that without any force applied, this is where the pendulum bob would rest.

The displacement of the pendulum bob is the arc length s. The weight $m\mathbf{g}$ has components $m\mathbf{g} \cos\theta$ along the string and $m\mathbf{g} \sin\theta$ tangent to the arc. Tension in the string exactly cancels the component $m\mathbf{g} \cos\theta$ parallel to the string. This leaves a *net* restoring force back toward the equilibrium position that runs tangent to the arc and equals $-m\mathbf{g} \sin\theta$.

For a simple pendulum, The period is $T = 2\pi\sqrt{\frac{L}{\mathbf{g}}}$.

The only things that affect the period of a simple pendulum are its length and the acceleration due to gravity. The period is completely independent of other factors, such as mass or amplitude. However, note that T does depend on **g**. This means that if we know the length of a pendulum, we can actually use it to measure gravity! This will come in useful in Figure 5.40.

TIPS FOR SUCCESS

Tension is represented by the variable **T**, and period is represented by the variable T. It is important not to confuse the two, since tension is a force and period is a length of time.

WORKED EXAMPLE

Measuring Acceleration due to Gravity: The Period of a Pendulum

What is the acceleration due to gravity in a region where a simple pendulum having a length 75.000 cm has a period of 1.7357 s?

Strategy

We are asked to find **g** given the period T and the length L of a pendulum. We can solve $T = 2\pi\sqrt{\frac{L}{\mathbf{g}}}$ for **g**, assuming that the angle of deflection is less than 15 degrees. Recall that when the angle of deflection is less than 15 degrees, the pendulum is considered to be in simple harmonic motion, allowing us to use this equation.

Solution

1. Square $T = 2\pi\sqrt{\frac{L}{\mathbf{g}}}$ and solve for g.

$$\mathbf{g} = 4\pi^2 \frac{L}{T^2}$$

2. Substitute known values into the new equation.

$$\mathbf{g} = 4\pi^2 \frac{0.75000 \text{ m}}{(1.7357 \text{ s})^2}$$

3. Calculate to find **g**.

$$\mathbf{g} = 9.8281 \text{ m/s}^2$$

Discussion

This method for determining **g** can be very accurate. This is why length and period are given to five digits in this example.

WORKED EXAMPLE

Hooke’s Law: How Stiff Are Car Springs?

What is the force constant for the suspension system of a car, like that shown in Figure 5.41, that settles 1.20 cm when an 80.0-kg person gets in?

Figure 5.41 A car in a parking lot. (exfordy, Flickr)

Strategy

Consider the car to be in its equilibrium position $\mathbf{x} = 0$ before the person gets in. The car then settles down 1.20 cm, which means it is displaced to a position $\mathbf{x} = -1.20\times10^{-2}$ m.

At that point, the springs supply a restoring force $\mathbf{F}$ equal to the person's weight

$\mathbf{w} = m\mathbf{g} = (80.0 \text{ kg})(9.80 \text{ m/s}^2) = 784 \text{ N}$. We take this force to be $\mathbf{F}$ in Hooke's law.

Knowing $\mathbf{F}$ and $\mathbf{x}$, we can then solve for the force constant $\mathbf{k}$.

Solution

Solve Hooke's law, $\mathbf{F} = -\mathbf{kx}$, for $\mathbf{k}$.

$$\mathbf{k} = \frac{\mathbf{F}}{\mathbf{x}}$$

Substitute known values and solve for $\mathbf{k}$.

$$\begin{aligned} \mathbf{k} &= \frac{-784 \text{ N}}{-1.20\times10^{-2} \text{ m}} \\ &= 6.53 \times 10^4 \text{ N/m} \end{aligned}$$

Discussion

Note that $\mathbf{F}$ and $\mathbf{x}$ have opposite signs because they are in opposite directions—the restoring force is up, and the displacement is down. Also, note that the car would oscillate up and down when the person got in, if it were not for the shock absorbers. Bouncing cars are a sure sign of bad shock absorbers.

Practice Problems

20. A force of 70 N applied to a spring causes it to be displaced by 0.3 m. What is the force constant of the spring?

a. -233 N/m
b. -21 N/m
c. 21 N/m
d. 233 N/m

21. What is the force constant for the suspension system of a car that settles 3.3 cm when a 65 kg person gets in?

a. $1.93 \times 10^4 \text{ N/m}$
b. $1.97 \times 10^3 \text{ N/m}$

c. 1.93×10^2 N/m
d. 1.97×10^1 N/m

Snap Lab

Finding Gravity Using a Simple Pendulum

Use a simple pendulum to find the acceleration due to gravity **g** in your home or classroom.

- 1 string
- 1 stopwatch
- 1 small dense object

1. Cut a piece of a string or dental floss so that it is about 1 m long.
2. Attach a small object of high density to the end of the string (for example, a metal nut or a car key).
3. Starting at an angle of less than 10 degrees, allow the pendulum to swing and measure the pendulum's period for 10 oscillations using a stopwatch.
4. Calculate **g**.

GRASP CHECK

How accurate is this measurement for g? How might it be improved?

a. Accuracy for value of g will increase with an increase in the mass of a dense object.
b. Accuracy for the value of g will increase with increase in the length of the pendulum.
c. The value of g will be more accurate if the angle of deflection is more than 15°.
d. The value of g will be more accurate if it maintains simple harmonic motion.

Check Your Understanding

22. What is deformation?
a. Deformation is the magnitude of the restoring force.
b. Deformation is the change in shape due to the application of force.
c. Deformation is the maximum force that can be applied on a spring.
d. Deformation is regaining the original shape upon the removal of an external force.

23. According to Hooke's law, what is deformation proportional to?
a. Force
b. Velocity
c. Displacement
d. Force constant

24. What are oscillations?
a. Motion resulting in small displacements
b. Motion which repeats itself periodically
c. Periodic, repetitive motion between two points
d. motion that is the opposite to the direction of the restoring force

25. True or False—Oscillations can occur without force.
a. True
b. False